Indefinite Divisibility

Jeff Sanford Russell

February 17, 2009

DRAFT: Please do not cite without permission.

1 Extensibility and divisibility

The *generality absolutist* holds that it makes sense to speak of absolutely everything there is. One of the main kinds of consideration brought against the absolutist is the argument from *indefinite extensibility*.[1] Abstractly, the argument goes like this: There is a sort of thing, F, such that, for any x's that are F's (or: for any "definite totality" of F's, or something along these lines), there is some further F, y, which is not one of the x's. So there are no things which comprise *all the F's*. Thus we cannot speak of absolutely everything, for whatever things that includes, it must include all the F's. Call a predicate like F *indefinitely extensible*.

The most popular substituend for F in the schematic argument is "set". It is natural to take the concept of a set to require that, whenever there are some x's, there is a set of just the x's; well-foundedness implies that this set is not itself among the x's. (We can do without this extra assumption by running the argument on the predicate "set that is not a member of itself" instead of mere "set".) Other candidates for this role are cardinals, ordinals, properties, and propositions. One feature all of these have in common is that they are abstract: their existence is doubted by people called *nominalists*. Someone who is inclined to deny that there are any F's (in the most serious sense of "there are"—the sense with which the generality-absolutist is concerned) will be untroubled by an indefinite extensibility argument concerning F's: she denies that any F's should be included in the inventory of absolutely everything. Thus it has been supposed that it is only the realist about sets, properties, etc., who is

I owe thanks to Andrew Bacon, Shieva Kleinschmidt, Colin Marshall, and especially Kit Fine for comments.

[1] Dummett, *The Seas of Language*, 441.

vulnerable to this particular line of attack from the generality relativist; the nominalist must be brought over by other considerations.[2]

Here I offer an indefinite extensibility argument concerning concrete objects, which should keep the nominalist absolutist up nights along with her platonist colleagues. But it is not only nominalist absolutism that is threatened by such an argument. For instance, a theorist might have supposed that the inconstancy of what there is is a peculiar feature of abstract objects. She might go on to say that, even though *there are* sets and properties, they are somehow *less real* than tables and chairs; she might then hold that we can speak absolutely generally about what *fully real* things there are, even though we can't for the shadow realm—so we can continue in absolutist innocence as long as we are doing our most serious metaphysics. My argument shows that this position is untenable.

One might doubt whether there could be such an argument. After all, it seems that there might be only finitely many concrete objects; in that case, surely there is no question of concrete objects being indefinitely extensible. Indefinite extensibility arguments purport that unrestricted quantification comes up short, very roughly speaking, because there are too many objects to quantify over. If objects are few, though, then surely unrestricted quantification will not fail for this reason. So indeed, the argument I propose does not show that concrete objects *actually* defy quantification; only that they *could*.

My argument is parallel to the challenge Ted Sider makes to Peter van Inwagen's mereological theory.[3] Van Inwagen claims that everything is either an atom or a living thing; Sider points out that insofar as we should think this thesis true at all, we should think it a necessary truth; moreover, it is incompatible with there being (non-living) "atomless gunk".[4] Since atomless gunk is a metaphysical possibility, van Inwagen's thesis is false.

Similarly, the proponents of absolute generality do not mean for us to believe that there are objects exhausting absolutely all there is *as a matter of mere contingency*. If this thesis is true at all, it is necessarily true. But then it should hold even in worlds containing unusual metaphysically possible objects. I will argue that there is a possible sort of concrete thing that satisfies the requirements of the indefinite extensibility argument: whenever there are some x's of that sort, there is a further thing of that sort not among the x's. Since the existence of any things of that sort is incompatible with absolutism, it follows that absolutism is at best contingently true—and so not true at all.

The possibility envisioned is (as in Sider's argument) one of a certain sort of

[2] E.g. Hellman, "Against 'Absolutely Everything'!"

[3] Sider, "Van Inwagen and the Possibility of Gunk"; van Inwagen, *Material Beings.*

[4] The term is coined by Lewis, *Parts of Classes*.

extremely divisible material object.[5] The extensible predicate F applies to the parts of some material object. That object is divisible in such a way that, whichever of its parts are taken into account, there will be still more parts not among them. Call such an object *indefinitely divisible*. Mere atomless gunk is not divisible enough to satisfy this criterion. A piece of ordinary atomless gunk may have only continuum-many parts; so few parts do not defy quantification (except for an extreme finitist). Rather, we must look to something that is more divisible yet.

Daniel Nolan raises the possibility of what he calls "hypergunk": an object that has arbitrarily large sets of parts.[6] He defends the view that hypergunk is metaphysically possible: it is logically consistent; its existence does not appear to be analytically false or a Kripke-Putnam kind of *a posteriori* impossibility; it can be clearly described in fairly natural terms; and "it may also be a reasonably natural way of spelling out a natural conception of unlimited divisibility".[7] He points out that some, such as C.S. Peirce, have apparently taken the space of the actual world to be something like this—and if there is any reason to deem this a mistake, it seems to be primarily on empirical considerations.

Hypergunk does not quite meet our demands. The trouble is that the definition of hypergunk is nominalistically unacceptable, since it is framed in terms of the cardinalities of sets. Perhaps the definition can be reframed to avoid any appeal to sets, but I don't know exactly how that would go; at any rate, I won't pursue it. But even though it is encumbered with set-theoretic baggage, Nolan's hypergunk can help carry us to a nominalistic argument. In fact, the kinds of indefinitely divisible object I propose can be shown (in the presence of set theory) to be species of hypergunk. Moreover, showing that (given set theory) an object is hypergunk is a way of showing it to be indefinitely divisible. That X is hypergunk is equivalent to there being injections from arbitrary sequences of ordinals into X's parts. This shows that X's parts are, so to speak, as extensible as the ordinals, a paradigm case of indefinite

---

[5] The idea arises from non-standard analysis and is suggested by Michael Potter:

> For the proposal now under consideration is that we should conceive of the continuum as indefinitely *divisible* in much the same way as the hierarchy is indefinitely extensible, and it seems inevitable that if this idea is thought through it will eventually lead us to abandon the idea that the continuum is a *set* of points at all. (Potter, *Set Theory and Its Philosophy: A Critical Introduction*, 146)

Daniel Nolan also alludes to this:

> If the mathematical realm can be said to be "indefinite" in size, should we say the same about the non-mathematical realm? (Nolan, "Classes, Worlds and Hypergunk.", 319)

[6] Ibid.

[7] Ibid., 307.

extensibility.[8] The nominalist, of course, shouldn't take such an argument to state the literal truth of the matter, since there are no ordinals (or injections, for that matter); nonetheless, she should still be convinced by the argument. For even though mathematical statements may not be *true*, the nominalist should still allow that mathematical reasoning is *conservative*, and so she should still be convinced by a mathematical argument from non-mathematical premises to a non-mathematical conclusion—even if the argument goes through mathematical intermediate steps such as attributions of hypergunkiness.[9] Thus I offer a different, nominalistically acceptable, characterization of a possible object that, in the presence of set theory, entails that it is hypergunk, and thus indefinitely divisible. This conclusion remains even after the set-theoretic scaffolding is stripped away.

Though it is intuitively useful, this scaffolding isn't essential: the conclusion can also be reached directly. First I should smooth some of the rough edges on my initial presentation. Though I use plural quantification in the intuitive presentation, officially I want to appeal to schemes: the kinds of object I consider satisfy schemes analogous to the Naïve Comprehension scheme of set theory:

(NC) There is some set y such that for each set x, $\phi(x)$ if and only if x is an element of y

(where ϕ is an arbitrary open formula). (NC) implies

(*) There is some set y such that for each set x, if $\phi(x)$ then $x \neq y$.

But (*) looks inconsistent: consider the instance where $\phi(x)$ is $x = x$. One might take this to show that Naïve Comprehension is false; the proponent of indefinite extensibility, though, argues that the apparent inconsistency is due to *ambiguity in the quantifiers*. In the statement of (NC), the initial "there is" is more inclusive than the subsequent "for each". (Alternatively, she might argue that classical logic does not here apply.)

How to make sense of this ambiguity, and how to make a rigorous general argument from a scheme of the form (*) to the denial of absolute generality are difficult questions; I don't address them in this paper. My goal is not to defend the indefinite extensibility argument. Rather, my conclusion is conditional: *if* there is a compelling argument from indefinite extensibility against absolutism concerning sets (ordinals, properties, etc.), then there is also a compelling argument involving concrete material objects. The characterization I offer is analogous to Naïve Comprehension in that it implies something of the same form as (*). Thus it faces the same *prima facie*

[8]This is defended as a sufficient condition for indefinite extensibility by Shapiro and Wright, "All Things Indefinitely Extensible", 258ff. They attribute it to Russell and Cantor.

[9]C.f. Field, *Science Without Numbers: A Defence of Nominalism*.

inconsistency as (NC), to be resolved in the same way as it is for (NC). Whatever the details of this resolution may be, it will involve giving up generality absolutism.

In fact, I give two such characterizations, one in terms of straight mereology, the other in terms of contact principles. Each characterization evidently describes a metaphysically possible sort of object: the conditions are easily statable, eminently intuitive, and logically consistent (as long as indefinite extensibility is itself consistent); in short, they pass all the usual tests of possibility. Each characterization entails that any object that satisfies it has indefinitely many parts: so if indeed it characterizes a possible object, then generality absolutism is not necessarily true.

2 Parts

Consider a piece of atomless gunk: if we restrict our quantifiers to range over its parts, we can affirm the following principle:

(G) Everything has a proper part.

It certainly seems like an object like this is a metaphysical possibility. Indeed, gunk has a long pedigree as a hypothesis about the make-up of *actual*-world objects. Anaxagoras writes:

> Nor is there a least of what is small, but there is always a smaller. . . .[10]

But it's not clear that mere atomlessness does justice to Anaxagoras's hypothesis. Here's one way to think about it. Starting from a world made of atoms, there are two ways to create a new, atomless world. The first is to banish the atoms, leaving the atomless composites behind. (To maintain consistency with standard mereology, we'll have to banish some other things too, such as finite fusions of atoms, but it is generally possible to do this in a way that still leaves some atomless objects behind.[11]) The other way is to *split* the atoms, introducing new objects to be their parts. I think it is a fair guess that the historical anti-atomists had a leap more like this in mind: from their peers' atomic conception of the world, we generate the appropriate atomless conception not by subtraction, but by addition—or better: by *division*. This is why they describe gunky objects as "infinitely divisible" (even though something made of infinitely many atoms is also infinitely divisible, strictly speaking)—it is contrasted with atomicity in that *more* divisions can be made, not fewer.

Say some things *form a chain* if each of them either is part of or has as a part each of the others. Now suppose there are objects forming a chain such that nothing

[10]Burnet, *Early Greek Philosophy*, §126; cited in Sider, "Van Inwagen and the Possibility of Gunk"

[11]See Russell, "The Structure of Gunk: Adventures in the Ontology of Space", 260ff.

is a proper part of all of them. (For this to be consistent with (G), the chain must be infinite.) In that case there is some other world that includes everything in the gunk world, but adds in as well an atom at the bottom of the chain, a proper part of each link of the chain.[12] So the original world is atomless by "subtraction" from the more inclusive world that includes an atom. The possibility I take Anaxagoras to envision, though, is that of atomlessness by *division*: the world should be more divisible than any similar atomic world. The anti-atomists' world is not what we might describe as an atomic world without the atoms, but rather one where all putative atoms can be *split*.

The following condition is equivalent to (G):

(G′) For any finitely many x's that form a chain, there is something that is a proper part of each x.

The preceding paragraph shows that what the proponent of serious divisibility really wants is a generalization of this, omitting the words "finitely many":

> For any x's that form a chain, there is something that is a proper part of each x.

I would call this condition "hypergunk", except that Nolan has already claimed that name for a condition that is not equivalent to it, so I will discontentedly call it "supergunk". (Nolan's hypergunk condition does not even entail atomlessness!) Schematically: let Chain $[\phi]$ abbreviate "For all x, y, if $\phi(x)$ and $\phi(y)$ then x is part of y or y is part of x."; then the supergunk condition is this:

> If Chain $[\phi]$, then there is some x such that for every y, if $\phi(y)$ then x is a proper part of y.

The supergunk condition (in the presence of set theory) entails the hypergunk condition. It is straightforward to construct injections from arbitrary ordinals into the parts of supergunk. (Let x_0 be any part. Let $X_\alpha = \left\{x_\beta : 0 \leq \beta < \alpha\right\}$. Given that the members of X_β form a chain for each $\beta < \alpha$, it immediately follows that the members of X_α form a chain. Then let x_α be a proper part of each member of X_α; $X_{\alpha+1}$ also forms a chain, so we can go on indefinitely.) It follows that any piece of supergunk is indefinitely divisible.

The supergunk condition is a plausible gloss on the intuitive notion of arbitrary divisibility. It satisfies Anaxagoras' condition that there is no "least of what is small"—not even an "ideal least", sitting at the bottom of some chain of diminishing parts. Thus supergunk is a way of making precise an account that has been

[12] The mathematical result alluded to here is that, given any Boolean algebra, one can construct an atomic Boolean algebra that includes it by treating equivalence classes of filters as "ideal points". (See Sikorski, *Boolean Algebras*, 29ff.)

thought by some to describe the *actual* composition of material things. It doesn't appear to be incoherent—unless the mere denial of absolute generality is incoherent.[13] The condition is logically consistent relative to extensible set theory (see the appendix). It looks like indefinite divisibility is genuinely possible. But in that case absolute generality is at best contingent.[14]

3 Contact

Consider an ordinary continuous object C—for simplicity, in one dimension.[15] Such an object is *dense*: the atomic parts of C (call them *points*) are ordered such that any two points x and y have a third point z strictly between them. Write $x \prec y$ for the ordering and for the moment restrict the quantifiers to points. Then the following is an equivalent formulation of the density principle:

> If there are finitely many y's such that each $y \prec x$, then there is some z such that for each y, $y \prec z \prec x$.

Now consider a generalization of this principle: the principle of *superdensity*:

(D+) If there are y's such that each $y \prec x$, then there is some z such that for each y, $y \prec z \prec x$.

This principle is the same as the density condition, except that the words "finitely many" have been dropped. The superdense continuum is indefinitely divisible. Suppose D is superdense, and a is a point in D that is not at the very end of D, so there is some $x \prec a$; then it follows immediately from the superdensity condition that the predicate "$x \prec a$" is indefinitely extensible.

We can generalize this idea to higher-dimensional objects and to objects that do not have point-parts. Besides parthood (which I will take to obey the axioms of

---

[13] As is claimed by Lewis, *Parts of Classes*, 68.

[14] One might object that supergunk is inconsistent on grounds that are neutral as to absolutism: Zorn's Lemma implies that there is a maximal chain of X's parts, and such a chain would violate the supergunk condition, since if anything is a proper part of each member of the chain, the original chain together with this new part would make a strictly larger chain. But in fact the same argument arises against *any* proposed instance of indefinite extensibility. Consider the ordinals: any chain of ordinals is an ordered sequence; its order type cannot be a member of that same sequence; so there is no maximal chain of ordinals. The reason for the apparent failure of Zorn's Lemma is that the lemma (like the comprehension scheme) involves an ambiguous quantifier (twice over, in stating the existence of a chain, and in stating that it is maximal). While we can truly say there is_1 a chain that is maximal_1 (i.e. no_1 chain properly contains it), we may yet go on to use a more inclusive quantifier "there is_2", with respect to which that chain is not maximal_2 after all—for there is_2 a chain that properly contains it. This is just what we should expect in the presence of indefinite extensibility.

[15] Are there any such things as "ordinary continuous objects"? Plausible candidates include fields and space-time regions; but at any rate, surely there *could have been* such things.

standard mereology), our mereotopological primitive is a binary relation of *connectedness*.[16] Intuitively, x and y are connected just in case they "touch" (including the case when they overlap). As a minimal set of constraints, this relation is reflexive, symmetric and monotonic (if x is connected to a part of y then x is connected to y), and it satisfies the following distribution principle:

> If x is connected to the fusion of y and z, then x is connected to y or x is connected to z.

As an example, take the regions of Euclidean space; it is natural to call two regions connected iff they are at zero-distance from one another (i.e., iff their closures intersect).

Now consider three additional constraints. Again we'll restrict our quantifiers to the parts of some particular object. First: there are at least two objects that don't touch (to avoid triviality). The second condition is a separation principle:

(Sep) If x and y are not connected, then there is some z that is not connected to either of them.

This is intuitive: if x and y do not touch, then there ought to be something between them that prevents them from touching.[17] This principle is satisfied in the Euclidean case with the "zero-distance" understanding of connection: if x and y are at positive distance from one another, then there is some region at positive distance from each: a point halfway between them, say. It also holds in the various gunky spaces I discuss elsewhere.[18] It is not a *necessary* principle: there might be disconnected space-times with nothing "holding them apart" but their own intrinsic topology. But it looks like a *possible* principle, perhaps even one that is actually true.[19]

The final condition is the infinitary generalization of distribution:

(CS) If x is connected to the fusion of the y's, then x is connected to at least one of the y's.

Or schematically:

(CS′) If x is connected to Fus $[y : \phi(y)]$, then x is connected to some y such that $\phi(y)$.

[16] See Clarke, "A Calculus of Individuals Based on 'Connection'"; Roeper, "Region-Based Topology", 255ff.; Russell, "The Structure of Gunk: Adventures in the Ontology of Space", 253ff.

[17] Compare Röper's (stronger) separability principle A10 (Roeper, "Region-Based Topology", 255).

[18] Russell, "The Structure of Gunk: Adventures in the Ontology of Space".

[19] Andrew Bacon has pointed out to me that it may be more natural to take regions in pointy topological spaces to be connected iff the closure of one overlaps the other. On this construal, the open intervals $(-\infty, 0)$ and $(0, \infty)$ do not touch, but every region in the real line touches one of them; so the separation principle doesn't hold in this context. So much the worse for the real line; the separation principle still holds in gunky spaces.

Call this the principle of *contact supervenience*: what touches the whole is determined by what touches the parts. This also has immediate intuitive appeal: for if x does not touch any one of some objects, then what could mediate its connection to the fusion? Of course, this principle is violated in the standard continuum (as well as in the models of gunk derived from the standard continuum): the interval $[-1, 0]$ is not connected to any of the intervals $[\frac{1}{n}, 1]$, though it is connected to their fusion.[20] But this fact is remarkable, and takes some getting used to. It would be much less surprising if contact supervenience were true.

In the one-dimensional context, with $x \prec y$ taken to imply that x and y are not mutually connected, the conjunction of general distribution and separation are equivalent to the superdensity condition (D+). So I will also call an object whose parts satisfy (Sep) and (CS) *superdense*.

Superdensity is compatible with the existence of atoms, but it is easier to get a grip on without them. For if a superdense object D is composed entirely of atoms, no two of which touch one another (i.e., there are no distinct adjacent points), it follows that no disjoint parts of D are connected to one another. (If x and y are disjoint, mutually connected, and decomposable into points, then x must have a point-part x_0 that is connected to y, and so y must have a point-part y_0 that is connected to x_0, even though x_0 and y_0 are distinct.) A pointy superdense object "falls apart", since disjoint pointy superdense objects cannot touch.[21] This disintegration does not infect gunky objects.

Moreover, anything superdense must be hypergunk. We can inductively define injections from arbitrary initial segments of the ordinals to sets of D's parts as follows. Let a and x_0 be mutually disconnected parts of D; for each ordinal α we can let x_α be some object that is disconnected from a and from the fusion of the x_β's for $0 \leq \beta < \alpha$; clearly x_α is distinct from each x_β.

We can also make an indefinite extensibility argument more directly, without the diversion into the ordinals, using the official schematic formulation (CS′). Restrict the quantifiers to the parts of D, and let a and b be things that don't touch. Let $x = \text{Fus}\,[y : \phi(y) \text{ and } y \text{ does not touch } a]$. By contact supervenience x does not touch a, so by separation there is some z such that z does not touch x (hence z is not part of

[20] John Hawthorne makes this point vividly. (Hawthorne, "Before-Effect and Zeno Causality.") Hawthorne argues that contact supervenience is false in worlds with a certain possible physics, and infers that it is *actually* false—so he may be taken to argue that it is *necessarily false*. But on inspection, his argument turns on topological facts about open and closed regions of physical space—facts which are evidently contingent. So the best way to take his argument is as showing that the principle of contact supervenience is false in a certain class of possible worlds, including both the toy-physics worlds he considers and also (probably enough) our own. But Hawthorne's argument does not show that contact supervenience is *necessarily* false.

[21] This is an extreme version of the contact problems raised by Zimmerman, "Could Extended Objects Be Made Out of Simple Parts? An Argument for 'Atomless Gunk'".

x) and z does not touch a. It follows that

> There is some non-a-toucher z such that for each non-a-toucher y, if $\phi(y)$ then $y \neq z$.

This shows that, like "set", "part of D that does not touch a" is an indefinitely extensible predicate.

Since the parts of a superdense object are indefinitely extensible, the absolutist must deny that there could be any such thing. But, aside from a commitment to absolutism, it is hard to see what would motivate this denial. The conjunction of the separation principle and the principle of contact supervenience seems like it could be true. It is logically consistent, as long as it is consistent to deny absolutism at all; it doesn't look like an analytic or necessary *a posteriori* falsehood; it is easy to describe and has the marks of clear conceivability. The main reasons we have for thinking *actual* continuous objects (such as space-time) are not superdense are empirical. If we had a well-developed and empirically adequate theory of continua that affirmed (Sep) and (CS), we might well adopt it. (In fact, mathematicians have made steps toward such a theory; see the appendix.)

As in the case of supergunk, we conclude that since superdensity is possible, absolutism is not necessary. But surely absolutism is not contingent; so it is in fact false.

4 Consequences for relativism

I have framed the divisibility argument as a challenge for the generality absolutist, but it also offers challenging lessons for the generality *relativist*. The possibility of indefinite multitudes of conreta closes off certain strategies for making sense of relativism.

First, the argument puts pressure against divided accounts that enlist "limited", inextensible predicates to play roles that extensible predicates cannot. For instance, Geoffrey Hellman takes a "sortalist" line, suggesting that even though there is something defective about "everything" and "every ordinal", nonetheless there is no obstacle to our using "every *donkey*" and the like.[22] This saves the apparent datum that "There are no talking donkeys" is perfectly intelligible: it says that "Every donkey is non-talking". He goes on to suggest that "flat-out" denials of existence, like "There are no ghosts", should be interpreted as a denial of ghosthood within some sufficiently broad, but still limited category: for instance, it might mean "Every space-time-occupant is a non-ghost".

[22] Hellman, "Against 'Absolutely Everything'!", 90ff.

The divisibility argument, though, shows that there aren't nearly as many limited predicates to go around as one might think. "Space-time occupant" won't do, for instance, since the parts of supergunk occupy space-time. Neither will Hellman's other suggestion, "cause", since presumably the parts of supergunk can enter into causal relations perfectly well. The problem runs deeper if we consider more peculiar kinds of supergunk. For instance, a donkey might have been made of supergunk; and it might even have turned out that among such a donkey's parts were donkey-*homunculi* (*asinunculi*?), and so on indefinitely. So it looks like "donkey" isn't a limited predicate either—and there won't be many limited predicates left for a divided strategy.[23]

Another relativist view that the division argument challenges is that the indefiniteness of existence depends on our ability to "construct" new objects. Kit Fine takes this kind of position: the indefiniteness of our quantifiers over (e.g.) sets consists in the fact that there is always what he calls the "postulational possibility" of there being further sets.[24] Given any use of the quantifier, one can make a stipulation that introduces a new, more inclusive use of the quantifier—for instance, by postulating a new set that includes every set the old use recognized.

Perhaps we can bring new sets into being by stipulative acts (and similarly ordinals and properties and propositions), but is it really plausible that we can stipulate new material objects into existence? If not, then Fine's story will not work in the face of indefinite divisibility. Indeed, Fine recognizes, "It is plausibly part of the meaning of "donkey" that donkeys cannot be introduced into the domain through postulation" (using this fact to explain how we can sensibly and categorically say, "There are no talking donkeys").[25] But even donkeys, it appears, can be divided indefinitely.

Perhaps, though, we have taken the divisibility argument the wrong way. Rather than showing that there are indefinitely extensible sorts of concrete things, perhaps the moral we should draw instead is that *parts* are *abstract* things. Taking a cue from the Aristotelian tradition, we may think of material objects not as already having constituent pieces, but instead of being divided merely "in thought"—parts (or at least some parts) are in some sense not *actual* but rather *potential*. If we say this about only *some* parts (so, for instance, our planet is really concrete despite being part of a solar system), then there is pressure to say *when* potential parts become actual. Once the details are supplied, we may be able to rehabilitate the divisibility argument by setting up the possibility so that the parts in question are actual after all (for instance, by imposing some qualitative pattern that distinguishes the parts from one another). Alternatively, we might succumb to the pressure to class *all* proper parts of the material universe as merely "potential", joining Spinoza and Jonathan

[23] Unless mere limitedness-in-*extension* is sufficient; but it is hard to see why it would be, if extensible quantificational expressions suffer from a deficiency of *meaning*.

[24] Fine, "Relatively Unrestricted Quantification".

[25] Ibid., 41–42.

Schaffer as monists.[26] But pursuing these ideas goes beyond the scope of this paper.

Appendix

In order to show that supergunk and superdensity are logically consistent—relative to indefinitely extensible set theory—and to give a more precise picture of what a world in which they held would be like, I can offer something like a model for these principles. Only "something like": I offer a (proper class) domain of objects, and interpretations of the predicates, such that, if we restrict our schemes to predicates with sets as their extensions (or if we restrict our plural quantifiers to range over sets), then the resulting interpretations of the supergunk and superdensity principles hold. Thus if Naïve Comprehension held in full generality (so any predicate whatsoever would have a set extension), then the supergunk and superdensity principles would hold with full generality in this "model". In whatever sense the Naïve Comprehension *does* hold in the "universe" of indefinitely extensible set theory, in that same sense the supergunk and superdensity principles should hold in the model.

Start with John Conway's "surreal numbers".[27] The surreal numbers and their ordering relation are defined inductively such that, given any sets L and R of surreal numbers such that each member of L is strictly less than each member of R, the pair $\langle L, R\rangle$ is a surreal number that lies strictly between the members of L and the members of R (See the references for details).

The surreal numbers are superdense in the first of the two senses, in terms of their ordering: if a is to the right of each member of a set of surreal numbers L, then $\langle L, \{a\}\rangle$ is a surreal number that lies strictly between them. But the surreal numbers are atomic (each individual number is an atom), which we saw implies that they fall apart: no surreal region is connected to any disjoint region.

What we want instead is "surreal gunk": an atomless space based on the surreal numbers. We can do this by analogy with Tarski's topological gunk, which has as its regions the open regular sets. Here is one way of doing it: let an *interval* be a pair of two surreal numbers $\langle x_1, x_2\rangle$ such that $x_1 < x_2$. An interval $x = \langle x_1, x_2\rangle$ is part of an interval $y = \langle y_1, y_2\rangle$ iff $y_1 \leq x_1 < x_2 \leq y_2$. The intervals x and y are connected iff $x_1 \leq y_2$ and $x_2 \geq y_1$. Let a *region* be a set of mutually disconnected intervals. We say a region X is part of a region Y iff each interval in X is part of some interval in Y; X and Y are connected iff some $x \in X$ is connected to some $y \in Y$. It's not hard to show that this satisfies the axioms of standard mereology and (connection-based) topology, and it is superdense and supergunky.

[26] Schaffer, "Monism: The Priority of the Whole".

[27] Conway, *On Numbers and Games*; Knuth, *Surreal Numbers*.

References

Burnet, John. *Early Greek Philosophy*. 3rd edition. London: A & C Black Ltd., 1920.

Clarke, Bowman. "A Calculus of Individuals Based on 'Connection'". *Notre Dame Journal of Formal Logic* 22, no. 3 (July 1981): 204–218.

Conway, John Horton. *On Numbers and Games*. 2nd ed. AK Peters, Ltd., 2000.

Dummett, Michael. *The Seas of Language*. New York: Oxford Univ Pr, 1993.

Field, Hartry H. *Science Without Numbers: A Defence of Nominalism*. Princeton Univ Press, 1980.

Fine, Kit. "Relatively Unrestricted Quantification". In *Absolute Generality*, edited by Agustín Rayo and Gabriel Uzquiano, 20–44. Oxford University Press, 2006.

Hawthorne, John. "Before-Effect and Zeno Causality." *Nous* 34, no. 4 (December 2000): 622.

Hellman, Geoffrey. "Against 'Absolutely Everything'!" In *Absolute Generality*, edited by Agustín Rayo and Gabriel Uzquiano, 75–97. Oxford University Press, 2006.

Knuth, Donald E. *Surreal Numbers*. Addison-Wesley Professional, 1974.

Lewis, David. *Parts of Classes*. Cambridge: Blackwell, 1991.

Nolan, Daniel. "Classes, Worlds and Hypergunk." *Monist* 87, no. 3 (July 2004): 303–321.

Potter, Michael. *Set Theory and Its Philosophy: A Critical Introduction*. Oxford University Press, 2004.

Roeper, Peter. "Region-Based Topology". *Journal of Philosophical Logic* 26, no. 3 (June 1997): 251–309.

Russell, Jeffrey Sanford. "The Structure of Gunk: Adventures in the Ontology of Space". In *Oxford Studies in Metaphysics*, edited by Dean Zimmerman, 248–274. Vol. 4. Oxford: Oxford University Press, 2008.

Schaffer, Jonathan. "Monism: The Priority of the Whole". December 2006.

Shapiro, Stewart and Crispin Wright. "All Things Indefinitely Extensible". In *Absolute Generality*, edited by Agustín Rayo and Gabriel Uzquiano, 255–304. Oxford University Press, 2006.

Sider, Ted. "Van Inwagen and the Possibility of Gunk". *Analysis* 53 (1993): 285–289.

Sikorski, Roman. *Boolean Algebras*. Berlin: Springer Verlag, 1964.

van Inwagen, Peter. *Material Beings.* Ithaca: Cornell Univ Pr, 1991.

Zimmerman, Dean. "Could Extended Objects Be Made Out of Simple Parts? An Argument for 'Atomless Gunk'". *Philosophy and Phenomenological Research* 56, no. 1 (March 1996): 1–29.